1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \left\langle t^2, 5t \right\rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

---

2. $\int x e^{x^2} \, dx =$

(A) $\frac{1}{2} e^{x^2} + C$ (B) $e^{x^2} + C$ (C) $x e^{x^2} + C$ (D) $\frac{1}{2} e^{2x} + C$ (E) $e^{2x} + C$

---

3. $\lim\limits_{x \to 0} \frac{\sin x \cos x}{x}$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{4}$ (E) nonexistent

4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

---

5. Which of the following gives the length of the path described by the parametric equations $x = \sin\left(t^3\right)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2\left(t^3\right) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2\left(t^3\right) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4 \cos^2\left(t^3\right) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos\left(t^3\right) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2\left(3t^2\right) + e^{10t}}\, dt$

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x = 2$.

II. f is continuous at $x = 2$.

III. f is differentiable at $x = 2$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

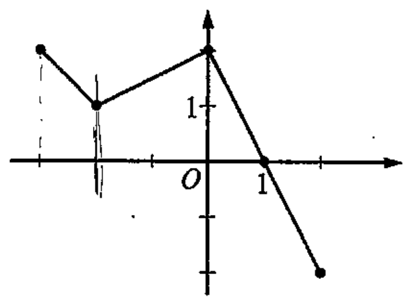
Given that $y(1) = -3$ and $\dfrac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

x	2	3	5	8	13
$f(x)$	6	–2	–1	3	9

8. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

). In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, 1)$?

(A) $-\frac{4}{3}$ (B) $-\frac{5}{4}$ (C) -1 (D) $-\frac{4}{5}$ (E) $-\frac{3}{4}$

1. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

12. Which of the following series converges for all real numbers x ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

---

13. $\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

---

15. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{1}{2\sqrt{e}}$ (D) $\frac{1}{\sqrt{e}}$ (E) $\frac{2}{\sqrt{e}}$

16. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

---

17. Let h be a differentiable function, and let f be the function defined by $f(x) = h(x^2 - 3)$. Which of the following is equal to $f'(2)$?

(A) $h'(1)$ (B) $4h'(1)$ (C) $4h'(2)$ (D) $h'(4)$ (E) $4h'(4)$

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k ?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

---

19. $\displaystyle\int \frac{7x}{(2x-3)(x+2)}\,dx =$

(A) $\frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

(B) $3\ln|2x-3| + 2\ln|x+2| + C$

(C) $3\ln|2x-3| - 2\ln|x+2| + C$

(D) $-\dfrac{6}{(2x-3)^2} - \dfrac{2}{(x+2)^2} + C$

(E) $-\dfrac{3}{(2x-3)^2} - \dfrac{2}{(x+2)^2} + C$

20. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$?

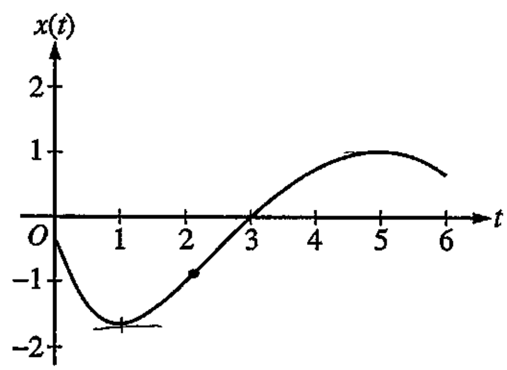
(A) $\ln 2$

(B) $\ln(1 + \ln 2)$

(C) 2

(D) e^2

(E) The series diverges.



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

x	0	1
$f(x)$	2	4
$f'(x)$	6	–3
$g(x)$	–4	3
$g'(x)$	2	–1

22. The table above gives values of f, f', g, and g' for selected values of x. If $\int_0^1 f'(x)g(x)\,dx = 5$, then $\int_0^1 f(x)g'(x)\,dx =$

(A) -14 (B) -13 (C) -2 (D) 7 (E) 15

---

23. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

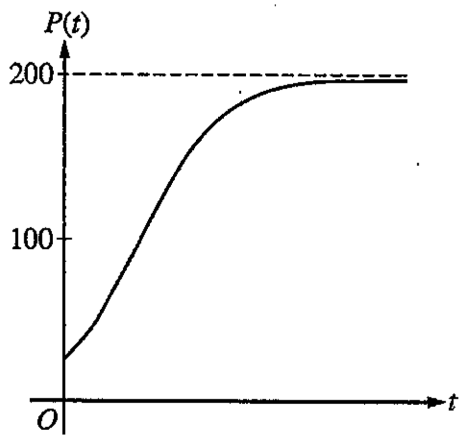
(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$



24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

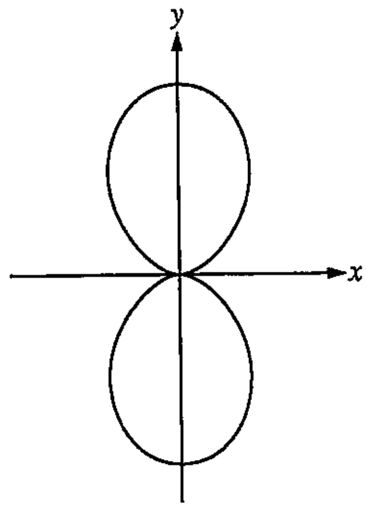
(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4



26. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta \, d\theta$

(B) $\int_0^{\pi} \sin^2\theta \, d\theta$

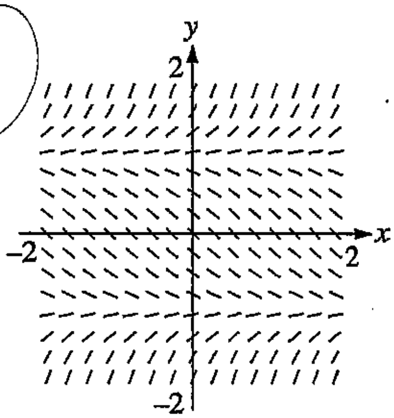
(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta \, d\theta$

(D) $\int_0^{\pi} \sin^4\theta \, d\theta$

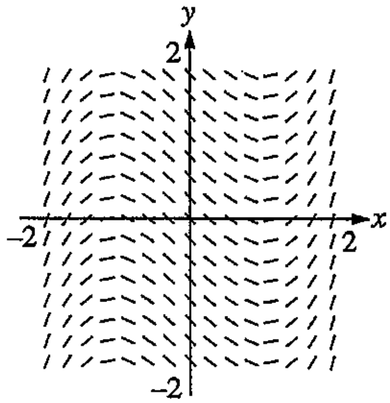
(E) $2\int_0^{\pi} \sin^4\theta \, d\theta$

27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

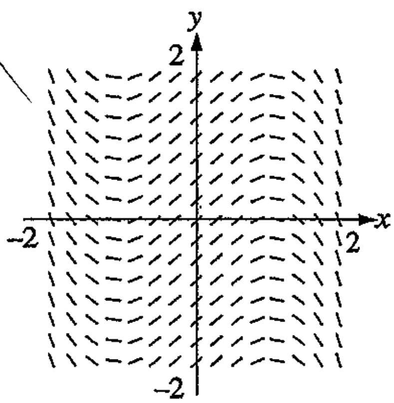
(A)



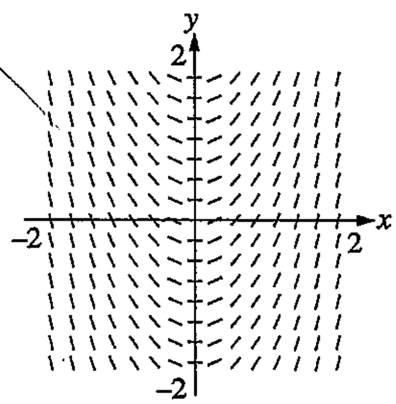
(B)



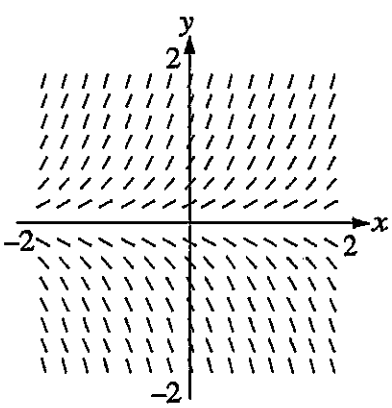
(C)



(D)



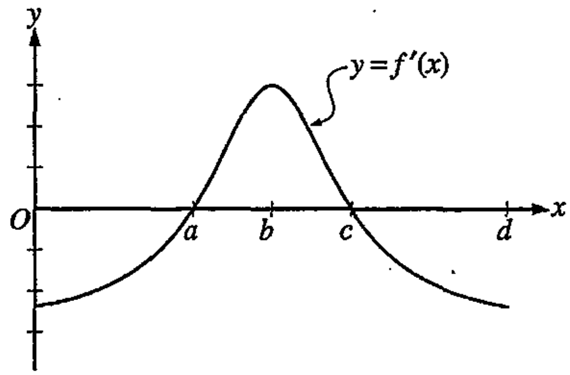
(E)



28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$

END OF PART A OF SECTION I



76. The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

(A) f has a local minimum at $x = c$.

(B) f has a local maximum at $x = b$.

(C) The graph of f has a point of inflection at $(a, f(a))$.

(D) The graph of f has a point of inflection at $(b, f(b))$.

(E) The graph of f is concave up on the open interval (c, d).

78

79.

---

77. Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\frac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

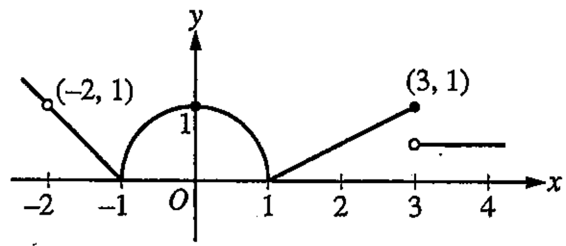
(A) 9.439 cubic meters

(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters



Graph of f

78. The graph of a function f is shown above. For which of the following values of c does $\lim_{x\to c} f(x) = 1$?

(A) 0 only

(B) 0 and 3 only

(C) -2 and 0 only

(D) -2 and 3 only

(E) -2, 0, and 3

---

79. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n\to\infty} a_n = k$

(B) $\int_1^n f(x)\,dx = k$

(C) $\int_1^{\infty} f(x)\,dx$ diverges.

(D) $\int_1^{\infty} f(x)\,dx$ converges.

(E) $\int_1^{\infty} f(x)\,dx = k$

80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

---

81. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

(A) $P - Q + R - S$

(B) $P - Q - R + S$

(C) $P - Q - R - S$

(D) $P + Q - R - S$

(E) $P + Q - R + S$

82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

---

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

84. Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

---

85. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from -3 to 3 ?

(A) Zero (B) One (C) Two (D) Three (E) Four

86. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

---

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$. Which of the following could be a table of values for g ?

(A)

x	$g(x)$
1	-2
2	0
3	1

(B)

x	$g(x)$
1	-2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	-2

(D)

x	$g(x)$
1	2
2	0
3	-1

(E)

x	$g(x)$
1	3
2	0
3	2

---

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
−1	−5	1	3	0
0	−2	0	1	1
1	0	−3	0	0.5
2	5	−1	5	2

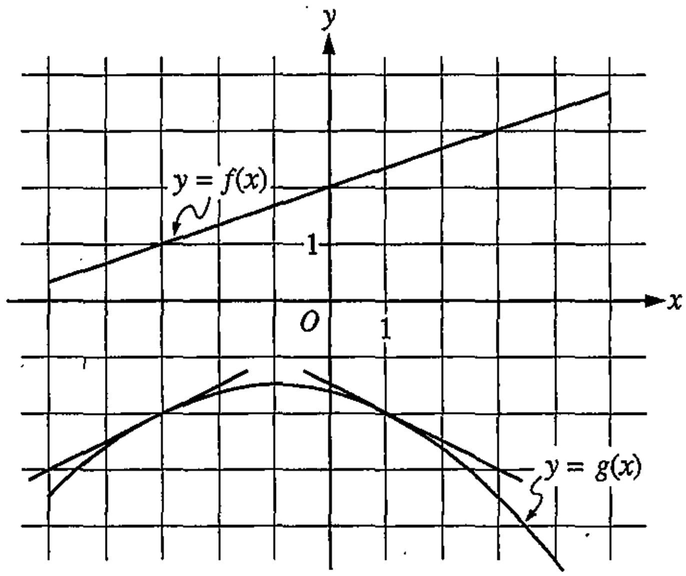
90. The table above gives values of the differentiable functions f and g and of their derivatives f' and g', at selected values of x. If $h(x) = f(g(x))$, what is the slope of the graph of h at $x = 2$?

(A) −10 (B) −6 (C) 5 (D) 6 (E) 10

---

91. Let f be the function given by $f(x) = \int_{1/3}^{x} \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \le x \le 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798 (B) 0.4 and 0.564 (C) 0.4 only (D) 0.461 (E) 0.999



92. The figure above shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.